The Tous Case study: mesh refinement & optimization data

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SUMMARY

In this project report a brief analysis of the numerical scheme used to simulate the flooding of a town, due to the failure of a dam is described. Special attention is put on the modelling and the analysis of the topographical data. All the data concerning to the flooding can be found in previous documents explaining the case study itself (see IMPACT Project Flood Propagation Case Study: The flooding of Sumacárcel after Tous Dam Break, by F. Alcrudo and J. Mulet).

1 INTRODUCTION

In two dimensional hydraulic free surface problems, when a mesh is required to be representative of some topography, as the number of cells used to create the mesh increases, the discrete representation of the real problem improves, so the accuracy of the results is enhanced, but the computing time grows and this can be a cumbersome difficulty. On the other hand, the experience says that, when the topography is smooth, larger cells can be used to simulate a flooding event but, if the topography is highly irregular, the number of cells must be increased in order to allow for a correct flow representation. One solution is to generate a mesh with local refinement, in order to reduce the calculation effort and, at the same time, be able to achieve the accuracy in the flow simulation results that a globally fine mesh would provide. When quadrilateral structured meshes are used, the refinement must be done in all cells contained in the rows and columns connected with the area whose cell-density we want to increase. Unstructured triangular meshes with local mesh refinement can lead to directional cell deformation depending on how they are created, and the grid refinement procedure is more complicated. The solution proposed here consists of using a structured triangular mesh of variable density following the variation of the bed slope. This produces a grid where the local refinement is introduced in the irregular topography zones, with a refinement depending on how irregular the topography is. Furthermore, the new cells are generated following a simple algorithm, without distortions and not affecting the size of the cells in the same row or column far from the area of interest.
This way, the study of the Tous dam-break problem is a perfect example in which topographic representation is a challenge for the proposed meshing.

2 MATHEMATICAL MODEL

The two-dimensional shallow water equations, which represent vertically averaged mass and momentum conservation, form a system with:

\[
\mathbf{U} = \left( h, q_x, q_y \right)^T
\]

\[
\mathbf{F} = \begin{pmatrix} q_x, \frac{q_x^2}{h} + \frac{gh^2}{2}, \frac{q_x q_y}{h} \\ \frac{q_y}{h} \\ \frac{q_y^2}{h} + \frac{gh^2}{2} \end{pmatrix}^T, \quad \mathbf{G} = \begin{pmatrix} q_x, \frac{q_x q_y}{h} \\ \frac{q_y}{h} \\ \frac{q_y^2}{h} + \frac{gh^2}{2} \end{pmatrix}^T
\]

where \( q_x = uh \) and \( q_y = vh \). The variable \( h \) represents the water depth, \( g \) is the acceleration of the gravity and \((u,v)\) are the averaged components of the velocity vector \( \mathbf{u} \) along the \( x \) and \( y \) coordinates respectively. The source terms in the momentum equations are the bed slopes and the friction losses along the two coordinate directions,

\[
\mathbf{S} = \begin{pmatrix} 0, gh(S_{0x} - S_{fx}), gh(S_{0y} - S_{fy}) \end{pmatrix}^T
\]

where,

\[
S_{0x} = -\frac{\partial z}{\partial x}, \quad S_{0y} = -\frac{\partial z}{\partial y}
\]

and the friction losses in terms of the Manning’s roughness coefficient, with

\[
S_{fx} = n^2 u \sqrt{u^2 + v^2} / h^{4/3}, \quad S_{fy} = n^2 v \sqrt{u^2 + v^2} / h^{4/3}
\]

The system has a Jacobian matrix, \( \mathbf{J} \), of the normal flux to each edge whose eigenvalues are a representation of the characteristic speeds.

The source term vector can also be decomposed in two different parts that will be treated separately: those based on spatial derivatives, such as the bottom variations \( \mathbf{B} \) and the rest, in our case the friction term \( \Gamma \), \( \mathbf{S}^* = \mathbf{B} + \Gamma \). An upwind approach has been adopted to model the bottom variations in order to ensure the best balance with the flux terms at least in steady state cases. The friction term \( \Gamma \) is discretized in a pointwise manner \( \Gamma_i^n = (\Gamma_i)^n \), so that the final expression for the numerical scheme is:

\[
\mathbf{U}_{i+1}^n = \mathbf{U}_i^n - \sum_{k=1}^{NE} \sum_{m=1}^3 ((\lambda^m - a^m - \beta^m^m - e^m) \mathbf{e}_k) \frac{d s_k}{A_i} \Delta t + \Delta t (\Gamma_i)^n
\]

The value of the conserved variables in each cell is updated using the surrounding information only if in the boundaries of the cell the waves have an in-going sense, when the value of the parameter \( CFL \) is less or equal one.
The most important problem when this test case is performed is related to the time step, proportional to the ratio between area and length of the cells in which the domain are discretized.

\[ \Delta t = CFL \Delta t_{\text{max}}, \quad CFL \leq 1 \]

where:

\[ \Delta t_{\text{max}} = \min_{i=1, N\text{CELL}} \left\{ \Delta t_{\text{max},i} \right\} \]

\[ \Delta t_{\text{max},i} = \min \left\{ \frac{\min\{A_i, A_{R_i}\}}{\lambda_k ds_k} \right\}_{k=1, NE} \]

where \( \lambda_k \) represent the eigenvalues in each edge \( k \), \( A_i \) is the cell area, \( ds_k \) is the length of edge, and \( NE \) is the number of edges in the cell.

3 TOPOGRAPHY MODELLING

This way the influence of the grid used on a 2D finite volume numerical model is evident. This is of particular interest when the flow pattern is complicated, but also when the bottom bathymetry is irregular and highly variable. In this work, the generation of new meshes is developed following the bathymetry characteristics in order to obtain a better approximation of the actual singularities of the valley geometry and to improve the accuracy of the results in the flow for those regions in which a more complex flow is associated to a more abrupt geometry. Using the proposed mesh refinement technique it is possible to generate locally refined areas without requiring a large time simulation in a simple way.

When flooding events are simulated over areas associated to small slope variation in space, large cells can be used to represent the bathymetry conforming a coarse grid. However, abrupt slope changes in space mean abrupt geometries and a higher influence of the terrain on the flow behaviour, so smaller cells are needed to represent both geometry and flooding. The criterion used in this work to decide the degree of refinement required is given by the variation of the slope in distance, that is, the gradient of bed slope.

4 MESH GENERATION

Quadrangular structured meshes have been used for many numerical purposes, and their refinement limitations are well known. In this paper, an unstructured triangular mesh is basically used, and, taking into account its connectivity properties, a local refinement is suggested in an easy way. The basic squared cell is divided by the diagonal in two cells, which are subdivided leading to four, eight, and sixteen cells. Four refinement degrees can be used as shown in Figure 1.
Levels 1 and 2 can connect perfectly their nodes to each other, and the same happens between Levels 3 and 4. However, connection cells must be supplied for any pair of Levels 1 or 2 with 3 or 4. Using connection cells a local refinement can be easily achieved, as in Figure 2. The first step of the refinement technique consists of dividing the whole domain in squared cells, of side length equal to the maximum length edge desired in the final triangular mesh, and so that each of the cells contains several topography data points. For each pair of data points within the cell \( i \) of the domain, the slope, \( S_{xy}(x,y) \) is calculated in both coordinate directions, \( x \) and \( y \). Once all the slopes are calculated, their maximum and minimum is searched. With this information the maximum slope gradient is stored for each cell.

Using the values of maximum gradient stored for each cell, the grid average maximum gradient slope can be defined, and the absolute maximum and minimum.

There is an important reason to limit the size of the initial cells used for slope searching to the same size as the largest edge of the final triangular mesh. If larger cells are adopted and if maximum refinement is required, the amount of refinement will be performed in a zone greater than necessary, increasing the final number of cells without an actual improvement of the bathymetry representation. Now, making use of the relative value of the maximum gradient in the cell, a refinement degree will be set. Although it seems natural to use a direct rule using the relative value of the maximum gradient, this criterion is not adequate. If the number of cells close to the absolute maximum is an important part of all the cells, an excessive refinement will be done; on the other hand, if the number of cells close to the absolute minimum slope gradient represents most of the cells, a poor refinement can be achieved.

In order to maintain the correct connectivity between cells it is necessary to generate the adequate connecting cells. The resulting grids always have a bigger number of
elements than the original one, which results in a smooth transition from non refined cells to maximum refinement cells.

In order to illustrate the performance of this meshing technique an example is presented. The test case consists of generating a suitable mesh for a scaled model of the Toce river valley, a watercourse of the occidental Alps, in Italy. The model scale factor is 1:100 and the approximate model dimensions are 50x11 m. The model reproduces the details of the real geometry. The bathymetry is supplied by means of a grid of points, with a typical distance of 5 cm. Figure 3 shows a view of the topography of the valley.

![Figure 3. Toce river valley: Bathymetry.](image)

Using these data to generate a uniform structured mesh, this results in a mesh with a total of 279530 cells, which is excessive if a small simulation time is required. Analysing the valley it can be seen that the bathymetry can be accurately represented with larger cells in some parts. A second attempt to use a uniform grid with cells 4 times the initial grid distance, 20 cm, leads to a dramatic reduction in the number of cells involved in the new mesh, but also some bathymetry details, like the river bed or the reservoir limits, clearly lose accuracy. In order to obtain a better result, the mesh algorithm proposed here is applied. Using a basic grid distance of 20 cm, a new final mesh with 56351 cells is generated. Figure 4 shows a detail of the reservoir using the initial density mesh of 5 cm. Figure 5 shows the same detail using a mesh with a constant density of 20 cm. On the other hand, Figure 6 shows the result when the meshing technique proposed here is used with an initial grid density of 20 cm, and Figure 7 the result using an initial grid density of 40 cm. In Figure 8, a wider region is displayed for the latter density.

Using local refinement, a good representation of the bathymetry is achieved in those places presenting a more complex geometry. For the Toce model good results are obtained when using local refinement over meshes with original cell edge of 20 and 40 cm.

![Figure 4. Detail of the reservoir. Constant density mesh, edge 5 cm.](image)
Figure 5. Detail of the reservoir. Constant density mesh, edge 20 cm.

Figure 6. Detail of the reservoir. Variable density mesh, initial edge 20 cm.

Figure 7. Detail of the reservoir. Variable density mesh, initial edge 40 cm.
Figure 8: Detail of the reservoir. Variable density mesh, initial density 40 cm.

5 APPLICATION TO THE TOUS DAM-BREAK

Doubt to the geometrical characteristics of the Tous case, the performance of a flooding simulation is impossible without the help of refinement techniques. In order to afford this test case the Sumacarcel geometry was represented initially by means of a variable mesh, where the length cell varies from almost 1 to 20 meters. As can be seen, in Figure 9 the streets were perfectly modelled, but the resulting computational time made the mesh useless.

Figure 9. Detail of Sumacarcel. Edge 1.25 m. Number of cells: 213689.
As second approach, a mesh where the size edge varies from 20 to 5 meters was generated, and although the geometry representation was not as accurate as in the previous one, the flooding simulation was performed in reasonable time.

![Figure 10. Detail of Sumacarcel. Maximum edge length 20 m. Minimum 5 m. Number of cells: 39231](image)

In order to enhance as much as possible the time step, and decrease the computing time, no refinement was done in the valley, and the refinement was restrained as much as possible in the zone close to the river bed (Figure 11)

![Figure 11. General overview of Sumacarcel.](image)
Finally, it is clear to see that the topographical approximation have a determinant effect on the flooding simulation. As Figure 12 shows, the water depth increases almost instantaneously in some gauge points, keeping their level constant during the drying phase. Those gauges are located in cells that, due to their geometry, finally act like pools. Other gauges never get wet, as the mesh approach the geometries as walls, doubt to the size cells.

![Figure 12. Numerical data stored in the gauge points.](image)

### 6 CONCLUSIONS

The main conclusion is that the development of local refinement techniques is fundamental to perform flooding simulation over gross areas, where highly irregular geometries are found, if not the computing time is unacceptable.

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8 REFERENCES


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