Experimental analysis of the general features of uniform mud-flow

A. Armanini, C. Dalri, L. Fraccarollo, M. Larcher & E. Zorzin
Dept. of Civil and Environmental Engineering, University of Trento, Italy

Keywords: uniform flow, mudflow, rheology, viscous diffusive model, USDP (Ultra Sound Doppler velocity Profiler)

ABSTRACT: New apparatuses were developed in the University of Trento to obtain mudflows in steady uniform conditions, in order to permit precise measurements of flow parameters. In a first experimental phase the natural material of the recent mudflow in South of Italy (Sarno, Campania) was employed, in a second phase a well-sorted porphyry material was utilized and in a third experimental phase small amounts of sand were added to the porphyry. Utilizing an ultrasound pulsed Doppler technique, velocity profiles of the steady-uniform mudflows were obtained in any position of the flow field. In particular conditions mudflow takes place in equilibrium with a bed constituted of the same material, showing a typically two-phase behaviour, characteristic of granular materials. The experimental profiles were compared with theoretical profiles for single-phase and double-phase mixtures.

1 INTRODUCTION

This work gives a general view of mudflows behaviour on the base of experimental and theoretical analysis.

The experiments were performed on three different types of mud. The first analyzed material is the natural mud of the recent mudflow in Sarno (Campania, Italy). The mud was circulated by means of a closed-circuit system consisting of a special conveyer belt realized in the University of Trento for granular debris flow. In uniform-flow conditions the material ran over a still layer of material at the bottom of the channel.

In a second experimental phase, a well-sorted porphyry material with particles characteristic diameter of 4 µm was utilized. This type of mud was studied with an innovative apparatus based on a pump able to operate with fluids having a specific weight up to 20,000 N/m³, and a properly designed device having the role of forcing the flux into the pump. The flow parameters were obtained using a pulsed Doppler ultrasound technique. A third experimental phase was carried out with mud obtained by mixing some sand to the porphyry mud and new velocity profiles were analyzed. Comparing the experimental velocity profiles with widely used theories for viscous flows permits to understand how they cannot explain the behavior of this kind of material.

2 THEORETICAL VISCOUS DIFFUSIVE MODEL

Mudflows are characterized by a granular solid phase and an interstitial fluid. The interstitial fluid is made of clear water or a mixture of water and fine solid particles, influencing with their electrochemical properties the rheological behaviour of the mixture. Due to the presence of highly viscous interstitial fluid, the Sarno mudflow can be classified as a “viscous debris flow” (Takahashi 1991).
Phillips et al. (1992) developed a theory for the dispersion of particles floating in a Newtonian fluid in laminar flow, which explains why in a mono-modal suspension of quasi-spherical particles the concentration distribution is non-uniform. The particle migration is due to gradients in the shear rate and concentration profiles.

Takahashi et al. (2000) tried to improve the theory of Phillips et al. (1992) by including gravity effect in order to apply it to viscous debris flow.

In this paper the Takahashi’s approach is adopted, whereas the rheology is assumed of a Bingham type, with the viscosity depending on the local solid fraction. The concentration and velocity profiles were obtained from the numerical solution of the resulting diffusive equation.

2.1 Stress tensor

The starting hypothesis for the formulation of the stress tensor derives from experimental observations (Krieger 1972) giving the expression of the viscosity η for a concentrated suspension, as reported in Equation 1:

\[
\eta = \eta(C) = \left(1 - \frac{C}{C^*}\right)^{-1.82}
\]

where \( C \) = volumetric solid fraction; \( \eta = \mu_c/\mu = \) relative viscosity; \( \mu_c = \) mixture viscosity; \( \mu = \) interstitial fluid viscosity; and \( C^* = \) maximum packing solid fraction. It is also assumed that the Bingham yield-stress depends only on the interstitial fluid characteristics.

2.2 Diffusive equation

There are hereafter considered the three different contributions governing the non-convective fluxes of particles through the flow depth, normally to the bed profile under uniform conditions.

The particle flux due to the variation of collisions frequency is given as (Phillips et al. 1992):

\[
N_c = -K_c a^2 \left(C^2 \nabla \gamma + C \nabla C\right)
\]

where \( K_c \) = dimensional constant experimentally determined; \( a = \) particle dimension; and \( \gamma = \) deformation velocity.

The particle flux due to the viscosity spatial variation is given as (Phillips et al. 1992):

\[
N_\mu = -K_\mu a \frac{d\gamma}{dz} \left(C^2 \frac{a^2}{\mu_a} \frac{d\mu_a}{dC} \nabla C\right)
\]

where \( K_\mu \) is an experimentally determined dimensional constant.

Solid particles tend also to deposit due to gravity. The associated flux, assuming the Stokes law, is given as (Takahashi et al. 2000):

\[
N_s = -\frac{2}{9} C \frac{d^2\left(\rho_s - \rho_f\right)g \cos \theta}{\mu} g(C)
\]

where \( \rho_s = \) coarse solid particles density; \( \rho_f = \) interstitial fluid density; \( \mu = \) interstitial fluid viscosity; \( g = \) gravity acceleration; \( \theta = \) equilibrium deposit slope; and \( g(C) = \) group sedimentation function due to the high concentration, that may be expressed as follows (Takahashi et al. 2000):

\[
g(C) = \frac{1-C}{\eta}
\]
Collecting all the above listed contributions, under stationary and uniform conditions the continuity equation for the solid phase can be written as follows (Takahashi et al. 2000):

\[
\frac{\partial C}{\partial t} = - \frac{\partial}{\partial z} \left( N_c + N \frac{\mu a}{\mu} + N_s \right)
\]

where \( t = \) time.

Figure 1. Geometrical sketch for the formulation of the viscous model.

Equation 6, in extended form, reads:

\[
K_i \left( C^1 \frac{d\gamma}{dz} + C^\gamma \frac{dC}{dz} \right) + K_{\mu C} \frac{d\mu_a}{dC} \frac{dC}{dz} + \frac{2}{g} C \left( \rho_s - \rho_f \right) g \cos \theta \frac{\theta \rho_s \mu}{\mu} g(C) = 0
\]

where \( z = \) height over the equilibrium deposit.

The applied shear stress at a general height is:

\[
\tau = \int_z^h \left( C \rho_s + (1 - C) \rho_f \right) g \sin \theta dz = \rho_f g h \sin \theta \left[ \frac{1 - z}{h} + \frac{h}{z} \int_z^h C dz \right]
\]

where \( \rho_s = \) coarse solid particles density; \( \rho_f = \) interstitial fluid density; \( g = \) gravity acceleration; \( \theta = \) equilibrium deposit slope; \( z = \) general height; \( h = \) surface level; and \( \Delta = \) relative density.

The binghamian shear stress at a general height is:

\[
\tau = \tau_y + \mu_a(C) \gamma
\]

where \( \tau_y = \) yield stress, which is independent from coarse particle concentration.

3 SOLID CONCENTRATION PROFILES

It is important to notice that the problem requires the assignment of what solid particles are part of the interstitial fluid, mixed with water, and what are contributing to the solid particle fraction \( C \).

In case of enhanced bimodal granulometric distribution, the separation is immediate; otherwise this point can be faced with accuracy only by experimental calibration. In this paper we will point out how it has been worked out.

The coarse particle concentration distribution function is obtained combining Equation 8 and 9 and substituting them in Equation 7.
\[
\frac{dC}{dZ} = \frac{-\frac{2}{9} \frac{\Delta}{K_c \tan \theta} (1-C) + C(1+\Delta C)}{\left[ (1-Z) + \Delta \right] \frac{\tau_y}{\rho_f gh \sin \theta} + 1 + 1.82 \left( \frac{K_{\mu}}{K_c} - 1 \right) \left( 1 - \frac{C}{C_s} \right)^{-1} \left( 1 - \frac{C}{C^*} \right)^{-1}}
\]

(10)

where \( Z \) = dimensionless height equal to \( z/h \) (Fig. 1).

If the yield-stress is zero, the coarse particle concentration distribution is similar to the results of Takahashi et al. (2000), Papa (2001), Dalrì & Zorzin (2002).

Equation 10 cannot be directly integrated. To solve the problem a further derivation along the \( Z \) direction is performed, and the following second-order non linear equation is obtained:

\[
\left( \frac{dC}{dZ} \right)^2 A + \frac{d^2 C}{dZ^2} B = 0
\]

(11)

where

\[
A = \left[ 1 + 1.82 \left( \frac{K_{\mu}}{K_c} - 1 \right) \frac{C}{C_s} \left( 1 - \frac{C}{C_s} \right)^{-1} \left( 1 - \frac{C}{C^*} \right)^{-1} \right]^2 \left( 1 - \epsilon C \right) - \left( \frac{2}{9} \frac{\epsilon}{K_c \tan \theta} + 1 + 2 \epsilon C \right).
\]

(12)

\[
B = \left[ 1 + \frac{C}{C_s} \left( 1 - \frac{C}{C_s} \right)^{-1} \right] - \frac{2}{9} \frac{\epsilon(1-C)}{K_c \tan \theta} + C(1+\epsilon C)
\]

(13)

Equation 11 can be analytically reduced to a first-order one (assuming \( dC/dZ \) is never null); the coarse particle equilibrium concentration, for a given channel slope at generic height, is obtained by the following integral:

\[
C = \int_{Z_{in}}^{Z_{out}} \frac{1}{\int_{Z_{in}}^{Z_{out}} A ds + \frac{1}{B} \frac{dC}{dZ}(Z_{in})} df + C(Z_{in})
\]

(14)

For the correct solution of the integral, two boundary conditions are needed.

The first one is the value of the concentration derivative at the bottom. The derivative at the bottom is a function of the slope and can be derived from Equation 10 and is reported in Equation 15. An analogy can be detected with the approach proposed by Rouse (1937).

\[
\frac{dC}{dZ} \equiv f \left( \frac{1}{\tan \theta} \right) \left( \frac{1}{1 + \Delta C_{ave} - \frac{\tau_y}{\rho_f gh \sin \theta}} \right)
\]

(15)
where \( C_{ave} \) is the depth-averaged solid concentration, as also defined in Equation 17. At this stage it is obvious that \( C_{ave} \) is not known a priori. This implies that the solution to Equation 15 can be obtained only by applying some iterative procedure in the numerical approach.

The second boundary condition is the bottom concentration and is assumed very close to \( C^* \).

The integration of Equation 11, with the proper boundary conditions at different slopes, gives the following results.

Figure 2. Total concentration variation along the dimensionless height at various slope angles.

Figure 2 shows that at the high slope-angles the flux due to diffusive forces is bigger than that one due to gravity, determining a higher concentration of coarse particles throughout the flow depth.

4 VELOCITY PROFILES

When the concentration profile is determined, the velocity profile can be derived from Equations 8 and 9. The following expression for the dimensionless form is obtained:

\[
\frac{\Delta}{1 + \frac{1}{s} C^{\mu}} \int_0^Z ds + \int_0^Z \frac{C ds}{1 + \frac{1}{s} C^{\mu}} = \frac{\tau_{y_\dim}}{\mu \left( 1 - \frac{C}{C^*} \right)}
\]

where \( u_{\dim} = u(z) \frac{\mu}{\rho_f gh^2 \tan \theta} \), \( \tau_{y_\dim} = \frac{\tau}{\rho_f gh \tan \theta} \).

The velocity profiles for different slopes are represented in Figure 3.
In Figure 3 can be observed that in the lower part the velocity gradients are small due to the high coarse-particles concentration, determining the formation of high viscosity forces; farther from the bottom a drop of concentration and a relevant fast increase of velocity are checked. The upper, constant part of the profile is due to being the yield stress exceeding the tangential external stress.

After determining the concentration profile, the average concentration $C_{\text{ave}}$ and the transport concentration $C_{\text{transp}}$ can be evaluated, as expressed in Equations 17 and 18.

\[
C_{\text{ave}} = \int_0^1 (C(Z) + C_{\text{fluid}}) dZ \tag{17}
\]

\[
C_{\text{transp}} = \int_0^1 (C(Z) + C_{\text{fluid}}) u_{\text{diml}} dZ \tag{18}
\]

where $Z$ is the dimensionless height equal to $z/h$ (Fig. 1); $C_{\text{ave}}$ = average concentration; $C_{\text{transp}}$ = transport concentration; $C_{\text{fluid}}$ = solid concentration in the interstitial fluid, assumed to be constant.

5 FIRST EXPERIMENTAL PHASE

The first part of the study examines the relation between transport concentration and equilibrium deposit slope for the natural material of the recent mudflow in Sarno. The natural material from Sarno (Italy) has the granulometric distribution shown in Figure 4a.

The mud was circulated by means of a closed-circuit system, consisting of a special conveyer belt realized in the University of Trento for granular debris flow. The circulation of the mud was also obtained with a special pump for slurry fluids. The conditions for the equilibrium bottom deposit were obtained by inserting a narrowing in the downstream part of the channel. In uniform-flow conditions the mixture runs over a still layer of material at the bottom of the channel.

According to Takahashi’s (1991) granular stability theory, the following expression is obtained:

\[
c_{\text{eqTak}} = \frac{\tan \theta}{\Delta(\tan \phi - \tan \theta)} \tag{19}
\]

where $c_{\text{eqTak}}$ = equilibrium concentration; $\phi$ = dynamic internal angle of friction.
Figure 4a,b. Granulometric distribution of the natural material from Sarno (Campania, Italy). Solid equilibrium concentration versus channel slopes.

Equation 19, based on Coulombian-type resistance-law, does not fit the experimental data, whereas the new developed theory (Eq. 18) does (Fig. 4b). For slopes less than 2°, the transport concentration is about constant and equal to the one of the interstitial fluid. This is the experimental information exploited to identify the solids belonging to the interstitial fluid.

6 SECOND EXPERIMENTAL PHASE: DOPPLER MEASUREMENTS AND THEIR INTERPRETATION

6.1 Experimental apparatus and Doppler technique

The experimental apparatus used in the second experimental phase is composed by a 5 m long and 35 cm wide rectangular flume. The walls and the bottom are made of Perspex to permit the observation of the motion. The flume can be inclined from 0 to 23° by means of an oleo-dynamic piston. The mud, obtained from well-sorted porphyry material and from a mixture of porphyry plus sand, was continuously recirculated by means of an appositely designed forced-flux pumping device.

The utilized DOP2000 ultrasonic velocimeter samples information about the local characteristics of the velocity field in uniform conditions; its working principle is based on the Doppler effect, detecting and processing the echoes of ultrasonic pulses reflected by particles in the flowing liquid.
In summary, a short train of sinusoidal waves with frequency $f_e$ is emitted from the transducer and then repeated at a lower frequency. Between these two pulses, the same transducer receives the waves reflected back by particles transported by the flow. The measurement of the travelling time to a certain depth or distance across the pipe gives the position of the scattering volume. By measuring the Doppler frequency shift $f_d$ at different times, it is possible to obtain an almost instantaneous velocity profile using the relation $v = c f_d / 2 f$, where $v$ is the velocity component in the direction of the ultrasonic beam, and $c$ is the speed of sound in the liquid. The resolution of this approach in distinguishing signals and velocity components is defined by the size and shape of the sampling volume and it is typically about a few cubic millimetres (Fig. 5) (Brito et al. 2001, Brunone et al. 2000).

![Ultrasound beam and sample volume](DOP 2000 User’s manual, revision 1.2). In the near field no measurements are taken.

In a few tens of milliseconds, a profile of fluid velocities is obtained. Only the velocity component parallel to the ultrasonic beam is accessible. The length of the profile is proportional to the listening time, which depends upon the pulse repetition frequency (PRF). Reliable velocities can only be obtained after a number of pulses have been shot and analysed.

The ultrasonic transducers are 0.5 MHz probes, 14 mm in diameter. The pulse usually consists of 8 cycles. The PRF is an essential parameter. The time $t_{PRF}$ between two emissions determines the length of the profile, but it also controls the velocity resolution. Indeed, if the time between two emissions is long, fast particles have moved too much to yield echoes that correlate. Mean velocity profiles are obtained by averaging a set of 1000 successive profiles.

6.2 Measurements method

The mud is obtained by adding water to the porphyry material (mean diameter 4 $\mu$m) and sand (mean diameter 500 $\mu$m). The resulting mixture is recirculated through the inclined channel and velocity measurements are taken from the side-wall along the spatial x-z planes, fixing y direction; from the bottom of the channel measurements are taken along the spatial y-z planes, fixing x direction. This is possible because the transducers are fixed on moving supports. The beams are inclined 20° in front of the wall surfaces, in order to get better velocity profiles along the flow direction and to reduce the sample volume and the interferences between flow portions at different velocities (Figs 6, 7a, b).

The measurements taken form the side-wall started from the surface level through the flow depth, with intervals of few millimetres, while measurements from the bottom were taken only from the middle of the channel because of some hindrance in positioning the transducer.

The sound celerity in the mud was previously measured by means of an appropriate experimental device.

The profiles were successfully checked from the side-wall and from the free-surface tracking some tracers by means of a high-speed digital video camera.
An example of mean velocity profile obtained from the bottom of the channel is reported in Figure 8 and it shows how measurements are slightly dispersed along the depth going from the bottom of the flow up to the free-surface.

Table 1. Settings for UDP.

<table>
<thead>
<tr>
<th></th>
<th>Figures 8 and 10</th>
<th>Figure 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dop2000 version</td>
<td>2.125</td>
<td></td>
</tr>
<tr>
<td>Emitting frequency</td>
<td>500 kHz</td>
<td>500 kHz</td>
</tr>
<tr>
<td>Emitting power</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Pulse repetition frequency</td>
<td>3521 Hz</td>
<td>5000 Hz</td>
</tr>
<tr>
<td>Burst length</td>
<td>8 cycles</td>
<td>8 cycles</td>
</tr>
<tr>
<td>Resolution</td>
<td>1.25 µs</td>
<td>2 µs</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Number of emission profiles</td>
<td>64</td>
<td>32</td>
</tr>
<tr>
<td>Doppler scalar factor</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Doppler angle</td>
<td>20°</td>
<td>20°</td>
</tr>
</tbody>
</table>
6.3 Interpretation of results

Figure 9a represents the profiles obtained from side-walls measurements taken every 1 or 2 millimetres in the upward direction. The measurements do not reach the bottom because the transducer support permitted the motion of the probe up to 2 cm above the bottom of the channel (Fig. 7a).

The mud used for this experiment was pure porphyry mud, with about 30% of total solid concentration. At every position of the probe, the velocity profile shows a peak near the side-wall, while it shows a constant value towards the middle of the channel. Since during experiments with pure water the velocity profiles does not show any peak, this phenomenon needs further investigation for a thorough comprehension.
The first hypothesis about the formation of the peak consists in the fact that the system does not warrantee enough NPSH (Net Positive Suction Head), so that the fluid tends to be filled with small bubbles. A second hypothesis is that while the mud is recirculated through the system, its temperature raises and some steam bubbles could remain trapped in the flow; this was checked by dipping a stick into the flow and observing the bubbles getting stuck on it. Bubbles absorb a lot of acoustic energy and could bias the measurements. Since the sound celerity depends on the concentration, a third hypothesis for the formation of the peak is related to the solid-concentration variation along the transverse direction of the flow, which is decrementing close to the side-walls.

The experimental measure of the concentration is based on a bulk measurement, therefore cannot take into account the variation of concentration along the transverse direction. The instrument utilizes a single value for the sound celerity and, since it is measured for the bulk concentration, when the concentration varies along the transverse, the measured velocity profile has a different value from the real one. If this latter hypothesis is right and if an analysis on velocity variation is performed, the variation of concentration from the middle to the external side of the flume is about 20%. This effect makes the study more complicated, because the local velocity measure is a function of the local sound celerity, while the experimental determination of the spatial position of the point where the measurement is carried out is a function of all the local celerity velocities between the investigated point and the transducer.

The vertical velocity profiles are shown in Figure 10. The profiles are obtained measuring directly the flow velocity putting the probe under the Perspex bed or enveloping the velocity results got through side-wall measurements. The origin of the axis system starts immediately after the near field. These experimental results examine the case of 10% of sand in volume added to the porphyry mud, with a maximum concentration about 60% in volume. The slope of the channel is 3.5°, the bulk concentration is 30%. The results were compared with Bingham and Herschel-Bulkley models in Figure 10, calculating the mechanical parameters of the models as a function of the solid concentration as suggested by O’Brien & Julien (1988) and Coussot (1997) and determining experimentally the dimensionless yield-stress for viscous-diffusive model as a ratio between the height of the plug and the total height of the fluid in movement.

![Comparison between different velocity profiles](image)

Figure 10. Comparison between theoretical velocity profiles and vertical velocity profiles obtained with bottom and side-wall measurements.

The experimental results show a good agreement between data obtained from different types of measurements. Herschel-Bulkley and Bingham rheological models cannot explain the behaviour of
artificial mudflow. Observing Figure 10 can be noticed that the lowest part of the velocity profile is close to zero near the bottom and it is followed by a high velocity variation along the depth.

The developed rheological model shows a good agreement with experimental results with an interstitial fluid concentration equal to 0.9 and a dimensionless yield stress equal to 0.595.

7 CONCLUSIONS

In particular conditions, mudflow has been observed to take place in equilibrium with a bed constituted of the same material. This happens using the natural material from Sarno (Italy) and the mixture of porphyry and sand, showing a typical two-phase behaviour. The new rheological model based on the viscous-diffusive theory interpolates the experimental measurements with a much higher accuracy than the classic Takahashi’s model (1991) based on the Coulombian resistance.

The application of the pulsed Doppler ultrasound technique allows a careful measure of the velocity profiles in the mudflow, even though some problems emerged. Some hypothesis were formulated in order to try to give a reasonable explanation to the velocity profiles peaks located near the flume side-walls, such as a variation of concentration along the transverse direction or the formation of bubbles trapped in the flow that may alter to the sound celerity, but further investigation is required for a thorough comprehension.

Up to now, the employed Doppler technology has some physical limits because it is not able to take into account that the sound celerity is a function of concentration.

ACKNOWLEDGEMENTS

Research work funded by CNR-G.N.D.C.I. (contract number 0000480 PF42) and by the European research project IMPACT.

REFERENCES