INTRODUCTION

In dam-break flow simulations, the topography of the valley is well known to have a significant influence on the results. The non-prismaticity of the valley cross section, the bed slope, the presence of bends inducing sudden changes in the flow direction are key factors affecting the flow.

In the shallow-water equations used to model dam-break flows, the terms related to the topographical effects appear as source terms. Special care is needed in their discretisation in numerical schemes to avoid erratic behaviour. For example, water at rest on an uneven channel bed might start to move, a non-constant discharge might result from the simulation of a steady flow over a bottom sill (Goutal and Maurel, 1997). Moreover, problems linked to the propagation of a dam-break wave on dry valley are worsened in the presence of bed slope.

Those issues are addressed by several authors. Bermudez and Vasquez (1994) proposed an upwind treatment of the source terms in a Roe-type finite-volume scheme. The approach is validated and extended to complex topographies by e.g. Garcia-Navarro and Vasquez (2000). The problem of water at rest on an uneven bed has been tackled among others by Nujic (1995). Capart et al. (1996) introduced a lateralised approach of the problem (the pressure terms in the fluxes are not the same for entering and exiting fluxes through a given interface, in such a way that the bottom-slope influence arises from this difference), while Mohanraj et al. (1999) proposed a characteristics-based treatment of the source terms. Several existing methods are extensively discussed and compared in Soares Frazão (2002), where an improved centred treatment of the bed slope is proposed.

This paper focuses on the effect of the bed slope using the example of a dam-break flow over a triangular shaped bottom sill. Experiments were carried out and interesting features of the flow could be identified and measured by using digital-imaging techniques. An improved centred treatment of the bed slope associated to a Roe-type finite-volume scheme (Soares Frazão, 2002) is presented and results from the numerical simulation are compared with the experiments.
on the flow. This latter series of experiments will be presented here.

![Figure 1. Experimental set-up and initial conditions, all dimensions in (m)](image)

This second series of experiments were simulated in a rectangular channel 5.6-m long and 0.5-m wide, with glass walls (Figure 1). The upstream reservoir extends over 2.39 m and is initially filled with 0.111 m of water at rest. The gate separating the reservoir from the channel can be pulled up rapidly in order to simulate an instantaneous dam break. Downstream from the gate, the channel is dry up to the bump. The symmetrical bump is 0.065 m high and has bed slopes of ±0.14. Downstream from the bump, a pool contains 0.025 m of water at rest, and a wall closes the downstream end of the channel. It is thus a closed system where water flows between the two reservoirs and is reflected against the bump and against the upstream and downstream walls.

### 2.2 Description of flow

High-speed CCD cameras were used to film the flow through the glass walls of the channel at a rate of 40 images per second. The experiments show a good reproducibility, allowing to combine the images obtained from different experiments to form a continuous water profile.

The flow is illustrated in Figure 2, showing images at $t = 1.8$ s, $t = 3.0$ s, $t = 3.7$ s, $t = 8.4$ s and $t = 15.5$ s, respectively. Only the downstream part of the channel, where the most interesting features appear, was filmed. The bump is indicated by a white line drawn on the images.

After the opening of the gate, the water flows on the dry channel and once reaching the bump, part of the wave is reflected and forms a negative bore travelling back in the upstream direction, while the other part moves up the bump, resulting in a wave propagation on an upward dry slope (Figure 2a, $x = 4.0$ m to 4.45 m). Then, after passing the top of the bump, the water flows on the downward dry slope until arriving in the pool of water at rest. There, the rapid front wave is slowed down abruptly and a positive bore forms (Figure 2b, $x = 5.2$ m). This bore reflects against the downstream wall and travels back to the bump (figure 2c, $x = 5.4$ m), but the water is unable to pass the crest. A second reflection against the downstream wall is needed to enable the wave to pass the bump and to travel back into the upstream direction (Figure 2d, $x = 4.5$ m). Multiple reflections of the flow occur both against the bump and the channel ends (Figure 2e).

![Figure 2: Experimental images of flow with bump marked as a white line](image)
2.3 Water level measurement technique

In order to measure continuous free-surface profiles, the flow was filmed through the glassy walls of the channel by means of a high-speed digital camera. By placing the camera at various locations along the channel, and then combining the images, it was possible to obtain surface profiles, as shown in Figure 2. The white line corresponds to the bed level and the water surface clearly appears. The position of the free surface is then measured by an automatic recognition procedure to each filmed image. This yields the result shown on Figure 3 for $t = 3.0$ s. Note that some zones are hidden by the vertical beams supporting the glass walls of the channel.

3 NUMERICAL SIMULATION

3.1 Finite-volume scheme

A shock-capturing finite-volume scheme solving the Saint-Venant shallow-water equations is used for the flow simulation. This numerical scheme should be able to reproduce the severe transient features of the flow such as the formation of bores, the strong reflections and the abrupt changes in wave-propagation speed. The one-dimensional equations written for a rectangular cross-section are recalled here, in vector form (Hirsch 1997)

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = \mathbf{S}$$

In Equation 1, $\mathbf{U}$ is the vector of hydraulic variables, $\mathbf{F}$ the flux vector and $\mathbf{S}$ the vector of source terms,

$$\mathbf{U} = \begin{pmatrix} h \\ q \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} q \\ \frac{q^2}{h} + gh^2/2 \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} 0 \\ gh(S_0 - S_f) \end{pmatrix}$$

where $h$ is the water depth, $q$ the discharge per unit width, $\sigma$ the so-called momentum flux, $S_0 = -\partial z_b/\partial x$ the bed slope and $S_f$ the friction source term, calculated for example by means of the Manning formula.

The finite-volume scheme, based on an integral form of Equation 1, reads (Hirsch 1997)

$$U_{i+1}^n = U_i^n - \Delta t \frac{F_i^{*} - F_{i-1/2}^{*}}{\Delta x} + S_i^n \Delta t$$

where $F_{i+1/2}^{*}$ and $F_{i-1/2}^{*}$ are the numerical fluxes at the cell interfaces (see Figure 4). Those fluxes are calculated by means of Roe's scheme (Glaister 1988)

$$F_{i+1/2}^{*} = \frac{1 + \tilde{F}_r}{2} F_i^n + \frac{1 - \tilde{F}_r}{2} F_{i+1}^n$$

$$+ \tilde{c} \frac{1 - \tilde{F}_r^2}{2} (U_i^n - U_{i+1}^n)$$

where the Roe averaged velocity $\tilde{u}$, water depth $\tilde{h}$, celerity $\tilde{c}$ and Froude number $\tilde{F}_r$, denoted with a tilde, are

$$\tilde{u} = \frac{u_i \sqrt{h_i} + u_{i+1} \sqrt{h_{i+1}}}{\sqrt{h_i} + \sqrt{h_{i+1}}}$$

$$\tilde{h} = \sqrt{h_i h_{i+1}}$$

$$\tilde{c} = \sqrt{g \frac{h_i + h_{i+1}}{2}}$$

$$\tilde{F}_r = \frac{\tilde{u}}{\tilde{c}}$$

The discretisation used is such that the bed level is known exactly at the cell interfaces instead of the cell centres (Figure 4). The mean bottom slope within the cell can thus be computed exactly. Also, the fluxes at the interface can be calculated exactly from the assumed constant water level in the two adjacent cells. Thus, computing the hydrostatic pressure term in each cell with the water depth at the interface ensures that the momentum terms are exactly balanced by the source term for a flow at rest, when
computed between two known interface bottom levels. This is also sketched in Figure 4: the topography is known at $p$ and the hydraulic variables $h$ and $q$ at $p$. Points noted with $p$ are interpolated linearly between the known interface points marked with $p$. Then, the bed slope source term is simply evaluated by a centred scheme

$$\left(ghS_0\right)_i = -gh_i \frac{z_{b,i+1/2} - z_{b,i-1/2}}{\Delta x} (6)$$

3.2 Difficulties posed by the bed slope

We have outlined that the numerical treatment of the source terms is of capital importance for a reliable simulation of the flow. In this bump test case, several difficulties arise that highlight the need of a proper discretisation of the bed slope and a well-posed treatment of the dry bed problem.

At first, there is a wave propagation on dry bed with an upward slope when the dam-break wave flows over the upstream slope of the bump. Then, the front wave flows downward on the downstream slope of the bump. Due to the discretisation of the problem, the bed slope is represented as a series of steps. When the water depth becomes smaller than the step height, the water profile becomes discontinuous, which might lead to numerical instabilities, as the water would have to "jump" from cell to cell rather than to continue flowing. To avoid this problem, a restriction is set on the numerical flux in such a way that no flux is transmitted through the interface $i+1/2$, i.e. $\hat{F}^{*}_{i+1/2} = 0$ if the water depth in cell $i$, $h_i$, is smaller then the step height $z_{b,i+1/2} - z_{b,i}$ (Figure 5).

In the downstream pool, located between the bump and the downstream end of the channel, the numerical difficulty is that the water has to stay at rest over the submerged part of the bump (see Figure 2a) before the arrival of the dam-break wave. This is a typical case of water at rest on a slope, where the flux balance of all cells should be zero in order to avoid spurious movements of the free surface. To ensure zero flux balance, all hydrostatic pressure terms, i.e. $gh^2/2$ in $\sigma = q^2/h + gh^2/2$, have to be in equilibrium with the bed slope $ghS_0$ (Figure 6). Care has thus to be taken especially for the last "wet" cell at the dry front on the beach (cell $i$).

![Figure 5](image1.png)  
**Figure 5. Restriction set on the numerical flux to avoid discontinuous surface profiles on slopes.**

Later, after the second reflection of the water against the downstream wall, when the water climbs up the downstream side of the bump and passes over its crest, another difficulty appears. A thin layer of water is still flowing on the downstream side of the bump, in the downstream direction, while the reflected wave climbs up the bump in the upstream direction. This problem is of the same type as the propagation on dry bed, as the water depth is very small, but it is increased by the abrupt change in velocity direction induced by the shock of two waves propagating in opposite directions.

Finally, the problem of drying of submerged cells recurrently arises for the cells located near the crest of the bump: it happens several times that after wave passage, the crest of the bump either emerges from the water, or is submerged again by the next wave, and so forth. Again, this problem is linked to the wave propagation on a dry bed, and highlighted by the fact that a definition for a dry cell is needed. In practice, a cell is considered as being dry when the water depth is below an arbitrarily fixed value $h_{min}$. By doing this, the generation of unrealistically high velocities, calculated as the ratio $u = q/h$ is avoided.

4 COMPARISON WITH EXPERIMENTAL MEASUREMENTS

This test case was computed on a 0.01-m mesh, in order to represent the bump as a series of steps with a sufficiently small height, compatible with the small water depth on the slopes. The results are first-order accurate in space, and were obtained with a CFL number of 0.9.
Comparisons with the experimental water-surface profiles are shown in Figure 7, for $t = 1.8\, \text{s}$, $t = 3.0\, \text{s}$, $t = 3.7\, \text{s}$ and $t = 8.4\, \text{s}$. At $t = 1.8\, \text{s}$ (Figure 7a), the front wave is running up the upstream slope of the bump. The water level is well reproduced by the numerical model, but the front wave propagates slightly too fast on the slope. In the downstream pool, the computed water profiles is perfectly horizontal, showing that the water stays well at rest, even on the slope. However, a discrepancy can be observed between the measured and computed water levels in the downstream pool, even though water there is at rest. This is due to an inaccurate measurement, as the calibration of the cameras was inaccurate when they were placed at the downstream end of the channel.

At time 3.0 s (Figure 7b), the bore formed by the reflection against the upstream slope of the bump is propagating back into the upstream direction. The numerical results are in agreement with the measurements, both for the water levels and the bore position. In the downstream pool, the bore formed at the arrival of the fast dam-break wave in the water at rest is also well reproduced by the numerical model. This latter bore then is reflected against the downstream wall (Figure 7c) and it appears that the numerical model overestimates this propagation speed. The more important discrepancies between numerical results and experiments are observed in the downstream pool, where complex processes occur, with multiple wave reflections. However, globally, the numerical scheme is able to reproduce these features.

At time 8.4 s, the bore formed by the second reflection against the downstream wall passes over the crest of the bump, in the upstream direction (Figure 7d). The computed water levels are in agreement with measurements, showing that this process is well captured by the numerical scheme.

5 CONCLUSION

In order to highlight the difficulties posed by the bed slope in a dam-break flow simulation, a test case was performed in a channel with a triangular bottom sill. Experiments were carried out, and by means of digital-imaging techniques, continuous surface profiles were acquired. Those allowed to identify and measure the key features of the flow, as well as the problems to be addressed for a reliable numerical simulation of this test case.

The proper representation of wave propagation on dry bed in the presence of either an upward or downward slope appears to be of capital importance. The problem of temporal stability for water at rest on a non-horizontal channel is also highlighted.

Computations were run with a Roe-type finite-volume scheme, featuring an improved centred treatment of the bed slope. The agreement with the experiments is good.

REFERENCES


